Appendix B. General equilibrium model 2

We consider another general equilibrium model (Model 2 henceforth) with an inseparable power utility function $u(c_t, \bar{n} - n_t) = (1/\delta)[c_t^{\eta}(\bar{n} - n_t)^{1-\eta}]^{\delta}$ and accommodating capital accumulation. This utility function has been used in Kydland and Prescott (1982) and Danthine, Donaldson, and Johnson (1998, DDJ hereafter). To enable the calibration, we presume two common conditions: (1) the risk-free asset is in zero net supply in all periods, i.e. $\{b_t\}_{t=1}^{T} \equiv 0$; and (2) the total shares of the stock are normalized to one in all periods, i.e. $\{s_t\}_{t=1}^{\infty} \equiv 1$,[29] and transfer the agent's problem into a Pareto optimum problem:

$$\begin{aligned} &\max_{k_{t+1}, n_t} E_t \left[\sum_{\tau=0}^{\infty} \beta^{\tau} (1/\delta)[c_{t+\tau}^{\eta}(\bar{n} - n_{t+\tau})^{1-\eta}]^{\delta} \right] \\ &s.t. \quad c_t + k_{t+1} = \alpha_0 {n_t}^{\alpha_1} k_t^{\alpha_2} {A_t}^{\alpha_3} \varepsilon_t + (1 - \Omega) k_t, \end{aligned} \tag{12}$$

where $\alpha_1 + \alpha_2 = 1$ and Ω denotes a constant depreciation rate. There is no adjustment cost in this model. It can be observed that the optimal investment and labor policies are mainly decided by the agent, which is intuitive as the agent is the shareholder of the firm.

Our numerical solution to Model 2 relies on the recursive value function iteration technique of Christiano (1990a, 1990b), which has been used by Danthine, Donaldson, and Mehra (1989, DDM hereafter) and DDJ (1998) to search for the optimal policy. Our first task is to deflate consumption and capital for stationarity like DDJ. One practical strategy is to set $\alpha_1 = \alpha_3$, which transforms the agent's maximization problem to

$$\begin{aligned} &\max_{\hat{k}_{t+1}, n_t} E_t \left[(1/\delta)[\hat{c}_t^{\eta}(\bar{n} - n_t)^{1-\eta}]^{\delta} + \sum_{\tau=1}^{\infty} (\prod_{s=1}^{\tau} \beta {\gamma_{t+s}}^{\eta\delta})(1/\delta)[\hat{c}_{t+\tau}^{\eta}(\bar{n} - n_{t+\tau})^{1-\eta}]^{\delta} \right] \\ &s.t. \quad \hat{c}_t + \hat{k}_{t+1} = \alpha_0 {n_t}^{\alpha_1} \hat{k}_t^{\alpha_2} \varepsilon_t + (1 - \Omega) \hat{k}_t, \end{aligned} \tag{13}$$

where $\hat{c}_t = c_t/A_t$, $\hat{k}_t = k_t/A_t$, $\hat{k}_{t+1} = k_{t+1}/A_t$, and $\gamma_{t+1} = A_{t+1}/A_t$.[30] Note that $\prod_{s=1}^{\tau} \beta {\gamma_{t+s}}^{\eta\delta}$ is the new discount factor now. We then apply the recursive value function iteration technique to identify optimal policies (choice variables), $\hat{k}_{t+1}$ and n_t, under different states (i.e. various combinations of state variables $\hat{k}_t$, γ_t, and ε_t). In each iteration i, the decision variables are $\hat{k}_i$ and n_i, and the state variables in each period include $\hat{k}$, γ, and ε (the time is fixed now). All feasible capital choice variable, $\hat{k}$, lie in a domain $S_k \equiv \{\hat{k}^1, \hat{k}^2, ..., \hat{k}^{lk}\}$, where $\hat{k}^1 < \hat{k}^2 < ... < \hat{k}^{lk}$ and $\hat{k}^{i+1} - \hat{k}^i = \hat{k}^i - \hat{k}^{i-1}$, and lk denotes the total number of capital choices. To calibrate the model, we simplify the technological growth γ as three values: γ_1, γ_2, and γ_3 with transition probability $\{p_{i,j}^{\xi}\}_{i,j=1,2,3}$. The non-technology shock $\varepsilon = \{\overline{\varepsilon}, \underline{\varepsilon}\}$ with probability $\{1/2, 1/2\}$.

[29] These two conditions do not affect the first order conditions in solving the n_t and k_{t+1}.

[30] $\max_{k_{t+1},n_t} E_t \left[\sum_{\tau=0}^{\infty} \beta^{\tau}(1/\delta)[c_{t+\tau}^{\eta}(\bar{n} - n_{t+\tau})^{1-\eta}]^{\delta}\right] = \max_{k_{t+1},n_t} E_t \left[\sum_{\tau=0}^{\infty} \beta^{\tau}(1/\delta)[c_{t+\tau}^{\eta}(\bar{n} - n_{t+\tau})^{1-\eta}]^{\delta}(A_{t+\tau}/A_{t+\tau})^{\eta\delta}\right]$ $= \max_{\hat{k}_{t+1},n_t} E_t \left[\sum_{\tau=0}^{\infty} \beta^{\tau}(1/\delta)[\hat{c}_{t+\tau}^{\eta}(\bar{n} - n_{t+\tau})^{1-\eta}]^{\delta}(A_{t+\tau})^{\eta\delta}\right]$. Note that $A_{t+\tau} = (\prod_{s=1}^{\tau} \gamma_{t+s})A_t$ and A_t term can be moved out of maximization equation.

The sequence of approximating value functions is illustrated by the following Bellman equation:

$$\begin{aligned} V_i(\hat{k},\gamma,\varepsilon) = & \max_{\{\hat{k}_i,n_i\}\in\Gamma} \{\frac{1}{\delta}[(\alpha_0 {n_i}^{\alpha_1}\hat{k}^{\alpha_2}\varepsilon + (1-\Omega)\hat{k} - \hat{k}_i)^{\eta}(\bar{n}-n_i)^{1-\eta}]^{\delta} \\ & +\beta\sum_{h=1}^{3}\sum_{j=1}^{2}\gamma_h^{\eta\delta}V_{i-1}(\hat{k}_i,\gamma_h,\varepsilon_j)p_h^{\xi}\frac{1}{2}\}, \qquad (14) \\ \text{subject to} \quad & 0 \le \hat{k}_i \le \alpha_0 {n_i}^{\alpha_1}\hat{k}^{\alpha_2}\varepsilon + (1-\Omega)\hat{k}, \\ & 0 \le n_i \le \bar{n}. \end{aligned}$$

V_i denotes the i-th iteration with current state variables $\hat{k}$, γ, and ε. h is the next state of γ, p_h^{ξ} denotes the transition probability between current state and next state of γ, and j is the next state of ε. Γ is the domain of feasible choice variables $\hat{k}_i$ and n_i ($\Gamma = \{S_k \times [0,\bar{n}]\}$).

In searching for optimal policies, DDM (1989) note that the second term on the right-hand side of equation (14) is irrelevant to choice of n_i. So, we can maximize the utility $u(\cdot)$ with respect to n_i first to find the optimal n_i by fixed point approximation. Then, we employ an exhaustive search to find the optimal $\hat{k}_i$ in S_k. The details of the search for optimal policies are available upon request.

Now we calibrate the model at the quarterly frequency and set $\beta = 0.98$, $\Omega = 0.05$, $\bar{n} = 1$, initial labor $n_0 = 0.3$, and initial capital $k_0 = 0.5$. There are 201 total possible capital levels (grid points) evenly distributed on the range between the minimum $k^1 = 0.01$ and maximum $k^{201} = 2.01$. The upper bound and lower bound are set so as to avoid the optimal choice occurring close to them. For the agent's utility function, we set $\eta = 0.333$ and $\delta = -0.1$ following Kydland and Prescott (1982). For the production function, we set $\alpha_0 = 15$, $\alpha_1 = 0.64$, $\alpha_2 = 0.36$, and $\alpha_3 = 0.64$. The value of α_0 is chosen to deliver: (1) output growth that approximates the historical data, and (2) positive dividend series. For technology growth, three possible values of γ are $[1, 1.005, 1.01]$ with probability $p_{i,i}^{\xi} = 0.5$ and $p_{i,j}^{\xi} = 0.25$ $(i \neq j)$. For non-technology shocks, we set $\overline{\varepsilon} = 1.005$ with probability 0.5 and $\underline{\varepsilon} = 0.995$ with probability 0.5.

We first randomly simulate $\{\varepsilon_t\}_{t=1}^{T}$ and $\{\gamma_t\}_{t=1}^{T}$ to initiate the time series dynamics of the economy. The state variable set in each period t, $\{\hat{k}_t, \gamma_t, \varepsilon_t\}$, can be computed according to these random processes. Then, the optimal policy set $\{\hat{k}_{t+1}, n_t\}_{t=1}^{T}$ can be identified with the searching procedure described in DDM (1989).

By adjusting for the deflator A_t, we obtain the output $\{F(n_t, k_t, A_t, \varepsilon_t)\}_{t=1}^{T}$, consumption $\{c_t\}_{t=1}^{T}$, and capital $\{k_t\}_{t=1}^{T}$. The time series of wages and dividends, $\{w_t\}_{t=1}^{T}$ and $\{d_t\}_{t=1}^{T}$, are decided by the firm's FOC: The equilibrium wage equals the marginal product of labor, and the dividend equals $F(n_t, k_t, A_t, \varepsilon_t) - k_{t+1} + (1-\Omega)k_t - n_t w_t$.

Given the pricing kernel, $m_t = \beta\frac{\partial u(c_t,\bar{n}-n_t)/\partial c_t}{\partial u(c_{t-1},\bar{n}-n_{t-1})/\partial c_{t-1}}$, we can compute the risk-free asset returns, $\{R_t^f\}_{t=1}^{T}$, according to $R_t^f = 1/E_{t-1}[m_t]$. We assume that the expected stock price is equal to the present value of total discounted dividends in 200 sequential quarters:

$$p_t = \sum_{\tau=1}^{200}\left[\left(\prod_{i=t+1}^{t+\tau} m_i\right) d_{t+\tau}\right]. \qquad (15)$$

So, we can derive realized returns $\{R_t^s\}_{t=1}^T$ according to $R_t^s = (p_t + d_t)/p_{t-1}$. This *ex post* approach assumes that realized returns equal expected returns on average.

Then, we generate 100 simulations, and each simulation is of length 1,000. We use all the variables in the period $t = 101$ to $t = 700$ to compute their means and standard errors. The averages of the means and standard errors of these simulations are reported in the following table. We first compare the results from this calibrated economy with the historical data, and our main target is to match the historical output growth. The mean and standard deviation of calibrated output growth are 0.005 and 0.008, both approximate historical data. The calibrated mean capital growth, consumption growth, and productivity growth are all 0.005, which are slightly lower than historical data. The calibrated mean risk-free asset return is greater than the historical data (0.013 vs. 0.006), while the calibrated mean stock return is smaller than the historical value (0.015 vs. 0.022). So, the market premium generated in this model is 0.002 per quarter.

Finally, we report the correlations between the current technology shock and output growth, the next period's stock return, and the next period's excess return in the bottom of the following table. We first find that the current technology shock is highly correlated with current output growth (88%), which verifies the fact that technology shock is the main driver of this economy. Second, we find that the stock return and premium in the next period are positively correlated with current technology shock (5% and 0.6%, respectively). So, we find market return/premium predictability from technology shocks in this economy as well.

	Calibrated Data		**Historical Data**	
Variables	Mean	Standard deviation	Mean	Standard deviation
Technological growth, γ_t	0.005	0.004	0.005	0.001
Output growth, $ln(F_t/F_{t-1})$	0.005	0.008	0.005	0.008
Capital growth, $ln(k_t/k_{t-1})$	0.005	0.004	0.007	0.003
Working time, n_t	0.243	0.000	0.289	0.005
Consumption growth, $ln(c_t/c_{t-1})$	0.005	0.009	0.006	0.006
Productivity growth, $ln(F_t/n_t) - ln(F_{t-1}/n_{t-1})$	0.005	0.008	0.006	0.007
Risk-free asset return, $R_t^f - 1$	0.013	0.004	0.006	0.006
Stock return, $R_t^s - 1$	0.015	0.004	0.022	0.083
$Corr(ln(F_t/F_{t-1}), \xi_t)$	0.883		0.029	
$Corr(R_{t+1}^s, \xi_t)$	0.050		0.231	
$Corr(R_{t+1}^s - R_{t+1}^f, \xi_t)$	0.006		0.239	

Notes: The sample period for historical data is 1977Q1–2004Q3. The technological growth is patent growth. The output is real GDP per capita. The capital is real capital per capita. The working time is the weekly working hours divided by total hours in five days. The consumption is real personal consumption expenditures (billion