Capacity Provision Networks: Foundations of Markets for Sharable Resources in Distributed Computational Economies *

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With the rapid growth of rich-media content over the Internet, content and service providers (SP) are increasingly facing the problem of managing their service resources cost-effectively while ensuring a high Quality of Service (QoS) delivery at the same time. To address this problem, the notion of Distributed Computational Economies (DCE), where infrastructure resources are traded, cooperatively shared and accessed through coordination mechanisms is gaining increasing attention. In this research, we conceptualize and model a DCE of Internet based storage provisioning for rich-media content delivery. This is modeled as a Capacity Provision Network (CPN) where participants possess service infrastructures and leverage their topographies to effectively serve specific customer segments. A CPN is a network of SPs coordinated through an allocation hub. We first develop the notion of discounted QoS capabilities of storage resources. We then investigate the spatio-temporal stability of the discount factors using a test-bed on the Internet through a longitudinal empirical study. Finally, we develop a market maker mechanism for optimal multilateral allocation and surplus sharing in a network. The proposed CPN is closely tied to two fundamental properties of Internet service technology: positive network externality among cooperating SPs and the convexity property of capacity allocation with geographically distributed service sites. We show that there exist significant incentives for SPs to engage in cooperative allocation and surplus sharing. We further demonstrate that intermediation can enhance the allocation effectiveness, and that the opportunity to allocation and surplus sharing can play an important role in infrastructure planning. In conclusion, this study demonstrates the practical business viability of a cooperative CPN market.

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1. Context and Motivation

Computational environments such as Grids (Foster et al. 1999), Peer-to-Peer (P2P) systems (Oram 2001) and a host of service networks such as Content Delivery Networks (CDN) (Rabinovich et al. 2001) and Internet Storage Infrastructures (ISI) (HTRC 2005) have emerged as flexible computing infrastructures that adaptively share distributed resources. The objective of flexible infrastructures is to leverage the topography of distributed resources to attain desired levels of performance within resource and cost constraints (Foster et al. 2000). This leads to the notion of Distributed Computational Economies (DCE) (Grimshaw and Wulf 1997, Waldspurger et al. 1992) built on such flexible infrastructures where resources can be traded, cooperatively shared and accessed through coordination mechanisms (Buyya 2002, Chun et al. 2001, Wolski et al. 2001). DCE signifies an evolution from high-end computing to decentralized service architectures where economic market mechanisms drive the trading of resources and services among globally distributed Service Providers (SP) in collectively servicing underlying market needs (Ferguson et al. 1996).

Extensive research as well as commercial developments in service-oriented grid-based DCE is rapidly evolving. Research test-beds such as iGrid (Brown et al. 1999), Distributed.net (http://www.distributed.net), SETI@Home (Sullivan et al. 1997), Condor Pool (Basney et al. 1999), GUSTO (http://www-fp.globus.org/testbeds), eGrid (Allen et al. 2001) and World-wide Grid (Buyya 2002) have been developed for research on collaborative computational resource-sharing as well as other organizational applications (Stockinger et al. 2001, Foster et al. 2001). Research on grid capacity trading and underlying economic incentives is emerging (Brooke et al. 2000, Frey et al. 2001, Sairamesh et al. 1998). Commercial computational grids with sharable resources have been developed by corporations
such as United Devices, Entropia, ProcessTree, Parabon and Popular Power. These commercial grids are based on fiscal incentives for individual resource owners to participate in trading in integrated network architectures (Buyya 2002). The underlying incentives to trade arise from the resource topography, demand and supply of resources, system integration, Quality of Service (QoS), system availability and scalability (Cocchi et al. 1993, Lalis et al. 2000).

In this research, we conceptualize and model a DCE comprised of Internet Storage Infrastructures. An ISI is a corporate storage solution including Storage Area Networks (SAN) and Network Attached Storage (NAS) (Preston 2002). Corporations are increasingly outsourcing these solutions to extend the reach and scalability of their operations and ensuring cost-effective levels of QoS. Two of the most popular current outsourcing solutions include Storage Service Providers (SSP) and ISI providers. While both SSP and ISI provide storage services and warehousing architectures for their clients, a major difference between them is that ISI provides in addition a global file system, geographical mirroring, and global load balancing so that its services are more aligned with the Internet with Internet-level scalability. Further, corporations are expected to spend over $3.5 billion in 2006 in these areas (HTRC 2005). Motivated by the growing market externalities for ISI services, increasing vertical integration among ISI consumers, and the consequential horizontal alignments among ISI providers, we conceptualize an ISI-DCE as Capacity Provision Networks (CPN) and develop the foundations of its market economics in this work.

2. Capacity Provision Networks

Two natural outcomes of the above DCE trends are leverage and resource trades. Leverage is obtained from the strategic positioning of a SP in an information supply chain; resource trades with others arise as a consequence. An SP derives this leverage from: (a) a geographic
advantage in service operations that others may not have, leading to enhanced QoS levels, and (b) the possession of necessary infrastructural/architectural capabilities to provide the service. Based on these factors, the notion of CPN in a DCE is introduced in Geng et al. (2003a, 2003b).

A CPN is a network of SPs that could trade and share computing resources in servicing their respective customers. While the CPN concept is aligned with the grid concept, it also represents different levels of conceptualization. At the business level, a CPN is a financial network of SPs with their business contracts defining the network arcs. At the service level, a CPN is a network of services with resource transfers between services defining the arcs. At the system level, a CPN is a network of system components with their specific technical linkages defining the arcs. The business level CPN is concerned with resource ownership, contracts and trades; the service level CPN is focused on resource leverage and QoS and the system level addresses the implementation. While a major part of the literature on grid computing is focused on the system level, a DCE is comprised of all the three levels and these have clear mappings to each other.

In this paper, we first model the business level CPN as a financial market for ISI services with a central market coordinator. We expect several innovative business models for Internet markets to originate from this concept. Second, we develop the service level leverages that drive the potential trades at the business level. Using standardized QoS as a basis, the topographical leverage of storage services is modeled as discounted QoS capabilities of storage resources. The discounted capacities characterize the efficiency gains that result from storage capacity trading among remote servers and provide a stable basis for the trades. Third, we investigate the spatio-temporal stability of the discount factors using a test-bed on the Internet
through a longitudinal empirical study. This study has established the discount factors to be very stable, highly resilient to indeterminate traffic behaviors over the Internet and possess useful concavity properties that can be effectively used to evaluate resources in potential trades. Furthermore, the traditional metrics for QoS are much more volatile and highly sensitive to Internet traffic pattern changes. In this regard, the proposed discount factors offer a stable and robust alternative. Fourth, we develop the economic foundations of the CPN and a market maker mechanism for trade coordination and surplus generation using a cooperative capacity sharing principle. Fifth, we develop a surplus sharing strategy using Shapley values. We develop the structural properties of the surplus sharing strategy and a sequential permutation sharing algorithm that asymptotically converges to the Shapley values. Finally we present detailed simulations with the CPN architecture.

In the CPN market, each unit of storage capacity is a two-attribute good: its value is determined by its price and its location on the Internet. The value of a CPN comes from the fact that Internet storage, as a technology, is characterized by positive network externality among storage sites: given the uncertainty in content demand, storage sites can better fulfill the demand collectively rather than separately. A CPN provides SPs not only a marketplace to trade, but also an incentive system for them to cooperate. Unlike classical mechanism design issues in economics where a proposed mechanism is not industry specific, the proposed design of the CPN market is closely tied to the characteristics of Internet storage technology, namely, positive network externality and sharability amongst geographically separated storage sites.

The organization of this paper is as follows. Section 3 presents the related work in the DCE context. Section 4 develops a topographical framework for service provisioning and the foundations of the capacity discounting concept. Section 5 develops the concepts of
topographical leverage and arbitrage in a CPN and presents the longitudinal study with
discount factors and the significant empirical conclusions. Section 6 presents the cooperative
capacity sharing principle and Section 7 develops the surplus sharing strategy. Section 8
presents the simulation experiments with results, and the conclusions are provided in Section 9.

3. Related Work

Extensive literature on resource management and sharing in DCE exists. A DCE is modeled to
be a network of resources with both local and distributed management with specific
ownerships. Consequently, the notions of producers and consumers of resources are widely
used in the grid and storage based DCE research community. Hierarchical as well as network
based resource management and sharing policies along with the economics of DCE are
outlined in (Buyya 2002). A DCE is modeled as comprising of a set of infrastructural elements
such as core middleware and application middleware with a set of architectural components
such as application portals, storage services and network services. Typically, most of these
components are sharable and hence, tradable. Consequently, resource management is central to
these processes and is studied under a variety of requirements such as system organization,
namespace, QoS, resource discovery, resource scheduling, state-space management and others.
A taxonomy of resource management systems is developed in (Krauter et al. 2002). This
taxonomy is based on several existing resource management systems such as AppLes (Berman
et al. 1997), Condor (Basney et al. 1999), Data Grid (Hoschek et al. 2000), Globus (Foster et
al. 1997), Javelin (Neary et al. 2000), Legion (Chapin et al. 1999), MOL (Gehring et al. 2000),
NetSolve (Casanova et al. 1997), Ninf (Nakada et al. 1999), Punch (Kapadia et al. 2000) and
Nimrod-G (Buyya 2002).
Several economic models of resource sharing that underlie the resource management processes have been proposed (Lazar et al. 1997, Huhns et al. 2000, Miller et al. 1998, Buyya 2002, Smith et al. 1980). These include well known models such as commodity markets, posted price, bargaining, contract-net, auctions, and proportional resource sharing to name a few. In addition, several research systems have also implemented many of these models. Some of the relevant systems include Mungi (Heiser et al. 1998), CSAR (Brooke et al. 2000), Mojo Nation (http://www.mojonation.net/), Stanford Peers (Cooper et al. 2002), Mariposa (Stonebraker et al. 1994), Spawn (Waldspurger et al. 1992), and GridSim (Buyya 2002). To our knowledge, none of these research works address the development of spatio-temporally stable surrogates for QoS in a DCE with a focus on topographical leverage and subsequently employing them in the design of efficient market mechanisms using surplus sharing strategies as developed in this paper. We briefly review the above systems as follows.


Market structures similar to the proposed CPN framework have been studied in other electronic market settings as well. Hendershott et al. (2000) study the impact of Electronic

4. A Topographical Framework for Service Provisioning

In this section, we develop a framework of analysis of the CPN. We first consider the topography of the service resources of SPs and determine the areas where the topography can be effectively leveraged to deliver a required QoS to customers. Second, we model the service-level agreements (SLA) between an SP and its customers and capacity-trading agreements among SPs in a CPN in a given time horizon. Using these models, we derive results on the efficiency of the CPN market and the incentives among CPN participants to cooperate.

4.1 Service Topography

We consider a set of SPs whose service infrastructures are networked via the Internet. Each SP represents the first point of access to capacity services for local clientele in its respective market and is represented by a CPN node. Each SP establishes an SLA with a corporation or any content provider. In the case of an SLA with a business, the SP provides a remote capacity service within the environment of a distributed data warehouse maintained by the business. Such service is common among Application Service Providers (ASP) such as Citrix Systems, USi and Storage Alliance where the master data set is held at the business headquarters and replications/partitions of the data sets are stored at the SP sites for faster access at remote sites. In the case of an SLA with content providers, a CDN-type arrangement usually exists, where copies of original content are stored in edge server farms maintained by the SP. In either case, when a user requests some data, it is served from a local edge server if it is available; otherwise, the data is retrieved from the master set at the origin server. If some of the requested
data is locally available, then only the missing data is retrieved from the origin server. We assume that the Internet is the medium for all these transactions and all the necessary applications are hosted at an edge server farm denoted by the corresponding CPN node.

We measure the QoS from an SP site by average delay, i.e. the average response time that a user experiences in accessing content. To illustrate, suppose that users need access to a total amount $\Omega$ of content. Further, suppose that the SP services an amount $\Omega_c \leq \Omega$ of the requested content locally, which can be accessed at average delay of $t_c$. The remaining content needs to be accessed from the origin via the Internet and let the average access delay be $t_0$. By definition, $t_0 > t_c$. For simplicity and without loss of generality, assume that the content is organized into a collection of data units where each unit is individually serviced and managed, and that each unit has the same probability of access by the users. Then, the users will experience an average delay of $t_1 = \frac{\Omega_c}{\Omega} t_c + \frac{\Omega - \Omega_c}{\Omega} t_0$ where $\Omega_c \leq \Omega$. Therefore the average end-user delay, $t_1$, is a non-increasing function of $\Omega_c$. In order to lower the access delay experienced by the users when $\Omega_c \leq \Omega$ the SP has two options: (a) to provide additional service capacity locally, or (b) to obtain the needed additional service capacity from another nearby SP. The viability of the second option depends on the cooperation agreement between the SPs and their relative proximity. The proximity between the SPs dictates the benefits of mutual trade as it results in the reduced performance problem: given a certain amount of local service capacity and same amount of remote service capacity, an SP would always prefer the local capacity since the performance of any remote capacity can be negatively affected by the delay between the two SPs. The farther the remote service, the more likely that retrieving data from it
could be delayed. This leads to the notion of *discounted remote capacity*, which is developed below.

### 4.2 Inter-SP Trades and Discounted Remote Capacity

Consider a different SP who can provide $\Omega_r$ units of content with an average access delay of $t_r$ to the SP discussed above. Call this new provider as the *remote SP*. Then by definition, $t_r > t_c$. Further, it should also be true that $t_r < t_0$, otherwise this remote service is of no value.

By employing the services of the remote SP, the average delay time is

$$
\frac{\Omega_c t_c + \Omega_r t_r + \Omega - \Omega_c - \Omega_r}{\Omega} t_0,
$$

which is lower than $t_1$. While an SP can lower the access delays for its consumers by availing of content services from another SP, the remote capacity is *discounted* as follows. Consider $t_i$ as before and set

$$
t_i = \frac{\Omega_c - S_c}{\Omega} t_r + \frac{S_c}{\Omega} t_r + \frac{\Omega - (\Omega_c - S_r) - S_r}{\Omega} t_0,
$$
to denote the same average delay experienced when both local and remote contents are used. In the latter expression, the local SP compensates for allocation of $S_c$ unit of local capacity to service content from its own site with allocation of $S_r$ units of servicing capacity from the remote SP. From the two expressions, we have

$$
S_c = \frac{t_0 - t_r}{t_0 - t_c} S_r.
$$

Since $(t_0 - t_r)/(t_0 - t_c) < 1$, it follows that remote service is not as effective as local service, which justifies the reduced performance problem. A unit of remote service only equals $(t_0 - t_r)/(t_0 - t_c)$ units of local service, which implies that remote service is *discounted* by a factor of $(t_0 - t_r)/(t_0 - t_c)$.

The reduced performance can be more conveniently modeled by using a *discount factor*. We define a discount factor $\delta_{ij}$ between SPs $i$ and $j$ if, from SP $j$’s perspective, one unit
of service capacity from SP $i$ is equivalent to $\delta_j$ units of SP $j$’s local capacity. If the average delay between these two SPs is $t_{ij}$, then, immediately we have $\delta_j = (t_0 - t_{ij})/(t_0 - t_c)$. The discount factors satisfy the following properties: $\delta_j \in (0,1]$ and $\delta_j = 1$. The definition of $\delta_j$ leads to the following properties.

**Marginal Discounting Property.** Consider SPs $i$, $j$, and $l$ such that $t_{il} = t_{ij} + t_{jl}$. Then, we have $\delta_{il} < \delta_j \delta_{jl}$.

The above property indicates that the marginal capacity discounting effect is larger for long delays than for short delays. Let $T$ be the space of all $t_{ij}$ and $D$ be the space of all $\delta_{ij}$ for all $i$ and $j$. Define function $\delta : T \rightarrow D$ according to the above definition of $\delta_{ij}$, i.e. $\delta_{ij} = \delta(t_{ij}) = (t_0 - t_{ij})/(t_0 - t_c)$. The definition of $\delta(t)$ logically leads to the following property.

**Concavity Property.** $\delta(t)$ is monotonically decreasing and concave with respect to all $t \in T$.

### 4.3 Structure of the SLA with Clients

Consider an SLA between an SP and its client. Let $\Omega$ denote the total volume of content that the client wants the SP to support during a specified contract period. Let the client also specify that content should be accessible at an average delay of no more than $t_1$ during this period. Assume that the actual demand for content during the contract period is unknown when the SLA is made and is stochastic. Let the continuous random variable $\Omega$ denote the content demand with a density function $g(\Omega)$ on $[0, \Omega]$. While the long-term content demand patterns are uncertain, and cannot be precisely specified at the contract time, the short-term demand for content can be accurately obtained. As a result, the whole contract-servicing span can be envisioned as comprising of a number of servicing periods. At each servicing period, let the SP
be provided with the exact demand for content for that servicing period. Clearly, the realized content demand would vary across the various servicing periods.

Now, consider the relationship between the demand for content and the required service capacity at any period of the whole contracting span. Given a demand for $\Omega$ content, the locally serviced content $\Omega_c$ required by the SP such that the content is delivered with an average access delay of $t_1$ (specified in the contract) is given by $\Omega_c = \Omega(t_0 - t_1)/(t_0 - t_c)$. Since $(t_0 - t_1)/(t_0 - t_c)$ is fixed, $\Omega_c$ is a linear function of $\Omega$ and follows the rescaled distribution of $\Omega$. Let $h(\Omega_c)$ and $H(\Omega_c)$ denote the probability density function and cumulative distribution function of $\Omega_c$, where $h(\Omega_c)$ can be obtained directly from $g(\Omega)$. Finally, let $\Omega_c$ denote the maximum content requirement locally serviced at any period of the whole contracting span, and thus $\frac{\Omega c}{\Omega} = \frac{\Omega(t_0 - t_1)}{(t_0 - t_c)}$.

Central to the structure of the SLA is the risk that arises from demand uncertainty. At one end of the risk-sharing spectrum is a contract where none of the risk is borne by the SP. In such a case, the SP simply agrees to provide a specified amount of capacity to service the content regardless of the average access delays for the duration of the contract at a fixed price. Consequences of any under-utilized capacity or lack of adequate capacity are borne completely by the client. In the case where the SP and the client share the risk, the contract specifies a service capacity amount $y$ ($y \leq \Omega_c$) and a corresponding penalty clause. In such a contract, the SP is committed to satisfying the demand of up to a service capacity of $y$ in each of the servicing period and would pay the client a unit penalty $b$ if for some reason this agreed capacity is not provided during any period of the contract span. The contract specifies a price $p$ that is paid to the SP for this service. The aggregated total penalty is computed at the end of
the whole contract span. Note that the risk borne by the SP is increasing in \( y \), and when \( y = \Omega \), all the risk arising from the uncertainty of content demand is transferred to the SP as the uncertainty of service demand. It follows then that \( p \) is strictly increasing in \( y \). For ease of analysis and exposition we will consider the case of \( y = \Omega \) in the following discussion. The results of this analysis can easily be extended to cases where the SP bears only some of the risk. Note that when the SP bears no risk, the problem faced by the SP is straightforward; either to provide the capacity at the agreed upon contract terms or to reject the contract.

In the next section we consider a network of multiple service providers. Such a scenario raises the possibility of topography-based intermediary services which result in complex capacity allocation and routing strategies among the SPs. We also present results from a longitudinal experimental analysis that provide strong support both for the viability of the discount factors for practical trade scenarios and for the presence of intermediation opportunities for capacity trading on the Internet.

5. Network of SPs: Arbitrage and Empirical Evidence

A network of more than two SPs can result in a more efficient allocation of capacity as each SP can provide and realize capacity to and from multiple SPs. The allocation patterns that emerge, however, could be potentially more complex as they provide opportunities for intermediation. We demonstrate this in the following.

5.1 Topographical Leverage and Arbitrage

We define an intermediary as an SP who could both provide and realize capacity in the servicing period. Such an intermediary could serve as a bridge for capacity allocation among other SPs. Consider SPs 1 and 2 that wish to engage in a capacity allocation. In the absence of an intermediary, the SPs would engage in allocation behavior as described in Section 4. Now
assume that a third SP (SP 3) that is strategically positioned so that \( t_{13} + t_{32} = t_{42} \). From the marginal discounting property it follows that \( \delta_{13} \delta_{32} > \delta_{12} \). By virtue of its strategic topographic location, SP 3 could facilitate a more effective capacity transfer without actually changing its local demand fulfillment in the allocation. Consider a situation where SP 1 provides 1 unit of capacity to SP 2. If a direct allocation occurs between the two, SP 2 realizes an effective capacity of \( \delta_{12} \) to service its clients. Now consider an alternate allocation strategy as follows: SP 1 provides capacity of 1 unit to SP 3; this is equivalent to \( \delta_{13} \) units of capacity for SP 3 at SP 3’s local site. Then SP 3 can allocate \( \delta_{13} \) units of its own local capacity for use by SP 2. Note that the capacity available for SP 3 is identical to the capacity available prior to the trade, and hence SP 3 is no worse off. However, the amount of capacity available for SP 2 is \( \delta_{13} \delta_{32} \) which is larger than when SP 3 is a non-participant in the capacity allocation. As a result, SP 3 realizes intermediary surplus sharing as long as it receives positive returns for providing the intermediary services. Such intermediation also enhances the overall effectiveness of the available capacity to provide services to the clients.

The above result can easily be extended to the general case of multiple intermediaries. Let \( G = (V, E) \) denote a network of \( N \) SPs, where each node \( i \in V \) represents an SP, and each edge \((i, j) \in E\) represents the segment connecting SPs \( i \) and \( j \). Let \( \delta_{ij} \) denote the discount factor associated with the edge \((i, j)\). We define a compound discount factor for each pair of SPs along each path connecting them in the network as follows. Consider any two SPs \( i \) and \( j \). Let \( (i, i_1, i_2, ..., i_j, j) \) denote a path between \( i \) and \( j \). Then, \( \delta(i, i_1, i_2, ..., i_j, j) \) is defined to be the compound discount factor for the path \((i, i_1, i_2, ..., i_j, j)\), and is the product of all the discount factors along this path. Let \( \delta_{ij}^\ast \) denote the maximum compound discount factor among all paths.
between $i$ and $j$. Accordingly, we term the path corresponding to the maximum factor $\delta_y^*$ as the maximal discount path between the two nodes. All nodes along the maximal discount path have intermediary opportunities in improving the allocation between SPs $i$ and $j$. The problem of determining the maximal compound discount factors between a pair of SPs is fairly straightforward. Using the inverse of discount factor $1/\delta_y$ along each edge $(i, j)$ of $G$, we solve a product-form of the shortest path problem.

The standard shortest path algorithm with a small modification will solve the problem. Instead of using the additive function in the shortest path recursion, the multiplicative function is used. Note that $0 < \delta_y \leq 1$, and hence, $1/\delta_y \geq 1 \forall i, j$. Therefore, the multiplicative recursion is positive and increasing with each computation of algorithm. As a result, the product-form of the shortest path algorithm will yield the optimal solution to the problem. Finally, it can easily be seen that the same path that solves the problem also yields the maximum compounded discount factor.

Successive allocation through intermediaries has strong implications for capacity allocation between SPs who are topographically far remote from each other. Typically, a consortium of intermediaries could generate such networks that are otherwise either too expensive to operate or even impossible. The behavior of the discount factors determines the intermediary options. Intermediary opportunities that arise due to topographical positioning have been observed in the related area of bandwidth trading in the telecom industry (Chiu et al. 2000).

5.2 Longitudinal Study of Discount Factors in a CPN

We have carried out a longitudinal experimental study of discount factors to assess their spatio-temporal stability and other properties in comparison with the direct measures of delay among
the service nodes in a CPN. Large-scale computer networks are often volatile, and any measure, if it is also volatile, poses significant implementation and monitoring problems. A logical question that follows is whether discount factors are more stable measures than actual time delays. More specifically, we address whether: (a) discount factors are less volatile than network delays, and (b) discount factors are spatio-temporally stable. Issue (b) implies whether the discount factors are statistically stable over the time-of-the-day and day-of-the-week parameters. Further, as addressed earlier, the concavity property is an important theoretical foundation of a CPN as it underscores the importance of intermediation to achieve optimal capacity allocation and sharing plans. In the empirical study, we were also motivated to seek out cases which exhibited strong concavity. This leads to the following: (c) whether concavity of discount factors holds true for some Internet based trading paths.

The design of the computational experiments entailed four sites, termed A (Austin, Texas), B (Buffalo, New York), C (Storrs, Connecticut), and S (Seattle, Washington). Note that the computation of a discount factor is based on the triplet (local site, origin site, remote site) with the constraint that the delay time from the remote site is lower than the delay time from the origin site. Thus the four experimental sites used in the study translated to 12 discount factor computations. The delay times required for the discount factor computation were obtained from the transfer of a data file with a random size of up to 10 MB between site pairs utilizing the file transfer protocol (ftp). Files transfers were conducted from every pair of sites on an hourly time interval for a period of 5 months in year 2004. This computational design yielded a total of 17,622 usable discount factors for analysis.

Table 1 reports the comparative volatility values (measured as the ratio of standard deviation over the mean) for delay times and discount factors. Except for the discount factor
with index 5, the results indicate that the discount factors are significantly more stable than network delays. Note that the higher volatility measure of the discount factor for index 5 results primarily from the low mean value (0.0487). For practical purposes, such low discount factors make it unviable for capacity sharing consideration, further strengthening the case for the use of discount factors for effective capacity sharing contracts.

The temporal stability of the discount factors was evaluated by the following empirical model.

\[
\delta_{ijk} = \alpha + \sum_{k=1}^{23} \beta_k \text{Hour} + \sum_{j=1}^{6} \gamma_j \text{Day} + \varepsilon_{ijk}
\]

where \(i\) indices the 12 distinct discount factors, \(j\) the day-of-week and \(k\) the time-of-day. The results, provided in Table 2, indicate that the discount factors are independent of hour of a day, or days in a work week (Monday to Friday). An adjusted \(R^2\) value of 0.978 and an \(F\) value of 18,819 indicate an excellent capture for the overall model. Finally, Figure 1 illustrates the evidence of concavity property when the origin site is located in Seattle and the local site in Buffalo. A capacity sharing agreement between the two sites that are intermediated via Connecticut can significantly enhance the overall sharing efficiency. Together the results strongly underscore the viability of discount factors as the appropriate measure for developing capacity sharing strategies.


While allocation according to maximal discount paths appears intuitive, it raises considerable amount of complexity in executing the allocations because of the capacity constraint that each SP faces, and in setting priorities when an intermediary is present in two or more maximal discount paths. In the following discussion, we present a Market Maker (MM) mechanism for optimal capacity allocation that is arbitrage-free. This mechanism provides socially optimal
allocation plans, and is applicable when all the service facilities are under the control of a single owner, or when the CPN hub generates allocation plans. In cases where there is no integrated planner, the following discussion provides an upper bound on the potential gains from capacity allocation, as measured by the overall reduction in the penalty paid by the participating SPs.

In the centralized mechanism, the SPs provide full information to the market maker and allocation decisions by the market maker are binding. The market maker’s objectives, illustrated in Figure 2, are two-fold: (a) to develop allocation plans that are globally optimal and (b) to develop surplus sharing plans that are fair to the participating SPs. The first objective is developed in the following discussion, and the subsequent objective of developing fair sharing plans is discussed below.

Consider a network $G = (V, E)$ of SPs. Let $s_i$ be the service capacity of SP $i$ and $y_i$ be the end-user demand at the contract servicing period. In terms of making its own capacity available for others, an SP would simply identify a portion of its own capacity for use by the other SPs. Let $S_{ij}$ denote the capacity made available by SP $i$ for use by SP $j$. To simplify notation, let $S_{ii}$ denote the capacity allocated by SP $i$ for its own use, and let $\delta_{ii} = 1$. Therefore it follows that $\sum_{j=1}^{N} S_{ij} = s_i \forall i \in V$. Similarly, the total capacity available for SP $i$ is given by $\sum_{j=1}^{N} \delta_{ji} S_{ji}$. Let $E$ denote the set of nodes with excess capacity (all $i$ where $s_i \geq y_i$), and $F$ denote the set of nodes with excess demand (all $i$ where $s_i < y_i$). Note that all the SPs in $E$ should never face a shortfall. Therefore, $\sum_{j=1}^{N} \delta_{ji} S_{ji} \geq y_i, \forall i \in E$. Similarly, for each node in $F$, effective capacity that is provided should not exceed what is required. Therefore,
\[ \sum_{j=1}^{N} \delta_{ji} S_{ji} \leq y_{i}, \forall i \in F. \]

The objective of the market maker is to minimize the total penalty incurred by the SPs. Suppose that \( b_{i} \) is the unit penalty at SP \( i \). The market maker solves the following formulation to generate the socially optimal allocation plan.

**Objective (MM)**

\[
\text{Minimize } \sum_{i \in F} b_{i} \left( y_{i} - \sum_{j=1}^{N} \delta_{ji} S_{ji} \right)
\]

subject to:

\[ \sum_{j=1}^{N} S_{ij} = s, \forall i \]
\[ \sum_{j=1}^{N} \delta_{ji} S_{ji} \leq y_{i}, \forall i \in F \]
\[ \sum_{j=1}^{N} \delta_{ji} S_{ji} \geq y_{i}, \forall i \in E \]
\[ S_{ij} \geq 0, \forall i \]

The above allocation capacity formulation can be solved as a linear programming (LP) problem and hence is computationally efficient. It can easily be verified that the solution to problem **MM** is arbitrage-free. We now turn to the issue of surplus sharing.

**7. Surplus Sharing**

In the context of a cooperative game, surplus allocation according to Shapley values are widely regarded as fair. The Shapley values in the context of a CPN can be defined as follows. Let \( N \) denote the set of players in this game (for exposition, we abuse the notation by using \( N \) to interchangeably denote a set of players or the total number of players in the set. In most cases the context will help clarify which concept is used). Let \( K \) be a subset of \( N \) and \( S_{ji}^{K}, i, j \in K \), be the optimal choices for the MM model computed on the subset \( K \) only. We define the characteristic function of Shapley values as:

\[
v(K) = \sum_{i \in F \cap K} b_{i} (y_{i} - s_{i}) - \sum_{i \in F \cap K} b_{i} \left( y_{i} - \sum_{j \in K} \delta_{ji} S_{ji}^{K} \right).
\]

This subset surplus is the maximal subset surplus that can be generated by this subset \( K \) only.
Consider $i, i \notin K$. SP $i$'s marginal contribution to this subset $K$ is $\varphi_i(K) = \nu(K \cup \{i\}) - \nu(K)$.

The probability that $i$ joins $K, i \notin K$, is $\frac{|K|!(|N| - |K| - 1)!}{|N|!}$. The Shapley value of $i$ in this cooperative game is the expected marginal contribution that $i$ can make by joining a subset $K, i \notin K$. Now formally define $\nu : N \rightarrow R^+$ as a side payment game where the set surplus is given by $\nu(K), \forall K \subseteq N$. Let $\phi = \{\phi_1, \ldots, \phi_N\}$ define how $\nu(N)$ is shared among $i = 1, \ldots, N$ according to Shapley values. We have

$$\phi_i(\nu) = \sum_{K \subseteq N \setminus \{i\}} \frac{|K|!(|N| - |K| - 1)!}{|N|!} [\nu(K \cup \{i\}) - \nu(K)].$$

It can be seen that $\sum_{i \in N} \phi_i(\nu) = \nu(N)$, which is the surplus of the global set to be shared among the $N$ members. The order of number of subsets $K \subseteq N \setminus \{i\}$ that $i$ can join is $O(2^{N-1})$. Each surplus computation $\nu(K)$ involves solving an LP problem. For each subset $K$ we need $\nu(K \cup \{i\})$ and $\nu(K)$ which requires 2 LPs for each computation of the marginal contribution $\varphi_i(K)$. The online supplement presents the theoretical analysis of the structural properties of the Shapley values for various special cases of the CPN. These properties lead to efficient solutions of the Shapley value computation problem in these scenarios. However, in the general case the direct computation of the Shapley values is of exponential order. We develop the following heuristic approach for the general case.

**7.1 Surplus Sharing Algorithm**

The computational burden in obtaining the Shapley values arises from the exponential number of permutations of the set of $N$ SPs that need to be considered. In the following discussion we develop a heuristic methodology that is based on sequential sampling from the exponential number of possible permutations. This heuristic strategy offers important advantages in
developing properties of the estimated Shapley values, and deriving stochastic bounds on the
Shapley value estimates. The algorithm is formally developed below.

**Algorithm: Sequential Permutation Sampling**

**Inputs:** $\delta, b_i, y_i, s_i$, for all $i, j \in N$

**Step 1: Initialization**

Select initial sample size $m$. Generate $m$ random permutations $\pi_1, \ldots, \pi_m$ of the set $N$
without replacement. For each permutation $\pi_q$ compute SP $i$’s marginal contribution
$\varphi_i(K(\pi_q,i)) = v(K(\pi_q,i) \cup \{i\}) - v(K(\pi_q,i))$ where $K(\pi_q,i)$ is the set of SPs that
precede SP $i$ in the permutation $\pi_q$. Compute the estimated Shapley value
$\hat{\phi}_{i,m}(v) = \sum_{q=1}^{m} \varphi_i(K(\pi_q,i)) / m$ and the estimated variance of marginal
contributions $\hat{\sigma}_{i,m}^2 = \sum_{q=1}^{m} (\varphi_i(K(\pi_q,i)) - \hat{\phi}_{i,m}(v))^2 / (m - 1)$.

**Step 2: Generate Permutation**

Generate an additional random permutation. Set $m \rightarrow m+1$ and compute $\hat{\phi}_{i,m+1}(v)$
and $\hat{\sigma}_{i,m+1}^2$.

**Step 3: Stopping Criterion**

If $\max \left( \left| \frac{\hat{\phi}_{i,m+1}(v) - \hat{\phi}_{i,m}(v)}{\hat{\phi}_{i,m}(v)} \right| , \left| \frac{\hat{\sigma}_{i,m+1}^2 - \hat{\sigma}_{i,m}^2}{\hat{\sigma}_{i,m}} \right| \right) > \varepsilon$, where $\varepsilon > 0$ is a pre-specified small
positive number, then go to Step 2.

**Step 4: Compute Shapely Values and Determine Bounds**

Set $m \rightarrow \bar{m}$. Use the normal distribution $N(\hat{\phi}_{i,m+1}(v), \hat{\sigma}_{i,m+1}^2 / \bar{m})$ to estimate the
Shapley values and the confidence intervals. STOP.
Note the above sequential permutation sampling algorithm requires solutions to $O(mN)$ LP problems. The estimated Shapley values thus obtained exhibit the properties of being unbiased, consistent, sufficient, asymptotically normally distributed, and asymptotically efficient. These properties are developed in the following analysis.

Let $\pi$ denote a permutation of the nodes. The $|N|$ permutations form a population $\Pi = \{\pi^1, \ldots, \pi^{|N|}\}$. Pick an arbitrary permutation $\pi^q$. Here a superscript denotes that the permutation is a member of the population set $\Pi$, while a subscript denotes that the permutation is a member of a sample set, denoted as $\Theta$. Then $i$ joins subset of $N \setminus \{i\}$ in $\pi^q$. Denote the subset as $K(\pi^q, i)$. $\phi_i(K(\pi^q, i))$ is the marginal contribution of $i$ to $K(\pi^q, i)$ in $\pi^q$. $\phi_i(K(\pi^q, i)) = v(K(\pi^q, i) \cup \{i\}) - v(K(\pi^q, i))$. Then $\phi_i(K(\pi^1, i)), \ldots, \phi_i(K(\pi^{|N|}, i))$ forms a population $\Pi_i = \{\phi_i(K(\pi^1, i)), \ldots, \phi_i(K(\pi^{|N|}, i))\}$, $\forall i = 1, \ldots, N$.

The sampling algorithm from the population is as follows. Take a randomly sampled set $\Theta$ of permutations without replacement from $\Pi = \{\pi^1, \ldots, \pi^{|N|}\}$, i.e. $\Theta = \{\pi_1, \ldots, \pi_m\}$. Compute $\phi_i(K(\pi_1, i)), \ldots, \phi_i(K(\pi_m, i))$, $\forall i = 1, \ldots, N$. Obtain the sample mean $\hat{\phi}_i(m)(v) = \sum_{q=1}^{m} \phi_i(K(\pi_q, i)) / m$, $\forall i = 1, \ldots, N$. Then $\hat{\phi}_i(m)(v)$ is an estimator of $\phi_i(v)$, which is the population mean. $\hat{\phi}_i(m)(v)$ is a sample mean. $\phi_i(v)$ is the population mean. (Green 2003) The sample mean is an unbiased, sufficient, consistent, asymptotically efficient estimator of the population mean. Let $\sigma_i^2$ be the variance for population $P_i = \{\phi_i(K(\pi^1, i)), \ldots, \phi_i(K(\pi^{|N|}, i))\}$. Then sample mean has the asymptotic distribution $\sqrt{m}(\hat{\phi}_i(m)(v) - \phi_i(v)) \xrightarrow{d} N(0, \sigma_i^2)$. The sample variance $\hat{\sigma}_i^2 = \sum_{q=1}^{m} (\phi_i(K(\pi_q, i)) - \hat{\phi}_i(m)(v))^2 / (m-1)$ is an unbiased and asymptotically efficient estimator for the population variance $\sigma_i^2$. The sample variance has the asymptotic distribution
of $\chi^2_{m-1}$. Therefore, when $m$ is large, we can use the sample mean $\hat{\phi}_{i,m}(v)$ to estimate the Shapley value $\phi_i(v)$.

Since the asymptotic distribution of both $\hat{\phi}_{i,m}(v)$ and $\hat{\sigma}^2_{i,m}$ depends on a large sample $m$, we need to determine whether $\hat{\phi}_{i,m}(v)$ and $\hat{\sigma}^2_{i,m}$ converge before using them to estimate the Shapley values. The stopping rule in the Sequential Sampling algorithm is used for this purpose. If

$$\max \left\{ \left| \frac{\hat{\phi}_{i,m+1}(v) - \hat{\phi}_{i,m}(v)}{\hat{\phi}_{i,m}(v)} \right|, \left| \frac{\hat{\sigma}^2_{i,m+1} - \hat{\sigma}^2_{i,m}}{\hat{\sigma}^2_{i,m}} \right| \right\} \leq \varepsilon,$$

where $\varepsilon > 0$ is a pre-specified small positive number, then we conclude that the estimated mean and variance converge. The value of the resulting $\bar{m}$ provides the requisite sample size.

8. Computational Analysis

We have conducted an extensive simulation analysis of capacity trading with the market maker mechanism. The key objectives of the simulation analysis is to provide critical insights on the surplus generation from capacity trading via CPN, and the performance of the sequential permutation sampling heuristic strategy for surplus allocation.

The design of the simulation analysis is as follows. The CPN is comprised on 100 SPs, each with a capacity of 0.5. The penalty for each of the SPs is generated from a uniform distribution on $(0.5, 1)$. The discount factor between each pair of SPs, $\delta_{ij}$, is generated from the uniform distribution on $(0,1)$. A hundred trading periods denoted as $\tau = 1,\ldots,100$ are simulated. In each trading period, the demand at each SP is drawn from a uniform distribution on $(0, 1)$. Note that the capacity at each SP is set at the expected value of the demand.

The first set of analysis focuses on the total surplus is generated from the market maker capacity allocation strategy. This surplus is generated from savings in the penalty payments by the SPs who face a shortfall in demand. We consider the impact of the number of
SPs participating in the CPN on the total surplus generated. Initially, one SP is selected randomly. At each iteration, one additional SP from the list of remaining unselected SPs is chosen randomly and added to the network, and the total surplus is recomputed with the additional SP. Figure 3 depicts the total surplus and the average surplus per SP, averaged over 100 trading periods, over the size of the network. As expected, the total surplus generated is non-decreasing in the size of the network. Interestingly, the average surplus per SP also exhibited a strong upward trend.

To further explore the impact by an SP as additional members join the network, we conducted the following analysis. A group of 5 SPs is selected as the control set. Additional members were iteratively added to the control set, and in each instance the actual and estimated Shapley values for each of 5 SPs is computed. Given the computational complexity inherent in the calculation of the exact Shapley values, these were computed only while the size of the network is no more than 10. The estimated Shapley values, however, were computed until all the SPs in the design set were included in the network. The sequential permutation sampling algorithm was used to estimate the Shapley values. This methodology resulted in two sets of data for each of the 5 SPs in the control set; one based on the true Shapley values and the other based on the estimated Shapley values. We then perform regression analysis on each of the 10 data sets, with the number of participating SPs in the network as the independent variable, and the Shapley values as the dependent variable. The estimated coefficients of the network size are all positive and significant at the 0.05 level. Figure 4 presents a histogram of the estimated coefficients. These results underscore the presence of strong positive network externality effects, whereby each of the SPs is benefited as new members join the network.
We now provide an analysis of the performance of our Sequential Permutation Sampling Algorithm. Note that the computational effort incurred in this algorithm depends on the sample size required to satisfy the stopping criteria. As described earlier, we consider trading networks of various sizes. For each trading network considered, the sampling strategy is repeated 5 times, for each of 100 trading periods. Figure 5 depicts the average values of the minimum, mean and maximum sample sizes required as a function of the network size. The figure reveals that the proposed sampling algorithm converges rapidly. The sample sizes required, even for large networks, is fairly small (all the sample sizes in the experimental analysis were under 500). This highlights the computational efficiency of the proposed algorithm for practical applications.

An important evaluation metric of the heuristic is the quality of the estimated Shapley values in relation to the true Shapley values. We provide this comparative analysis for networks ranging in size from 5 to 10 SPs, repeated over 100 trading periods. We compute the metric “Coefficient Variances” which is defined as the ratio of the standard deviation of the estimated Shapley values to the true Shapley values. A histogram of the “Coefficient Variances” is shown in Figure 6. Vast majority of the estimated Shapley values are within 20% of the true Shapley values.

For larger networks, the computational burden involved in obtaining the true Shapley values precludes a direct comparison between the exact and estimated values. For a large network of 85 SPs, we compute the ratio of the standard deviation to the mean for each SP, based on 5 samples. A histogram of the computed ratio is shown in Figure 7, and it suggests that the estimated Shapley values are relatively impervious to the sample that is drawn.
While the estimated Shapely values may deviate from the true Shapley value for an SP in a given trade scenario, we are motivated to consider this deviation from a cumulative perspective. In other words, capacity trading is expected to occur repeatedly over time and a well-designed surplus sharing mechanism would ensure convergence in cumulative surplus allocations. Let $\phi_i^\tau - \hat{\phi}_i^\tau$ denote the difference between the actual and estimated Shapley values for SP $i$ in a trade that occurs in period $\tau$. The cumulative percentage deviation of the estimated Shapley values is given by $\left( \sum_{i=1}^N \sum_{\tau=1}^\Gamma (\phi_i^\tau - \hat{\phi}_i^\tau) \right) / \left( \sum_{i=1}^N \sum_{\tau=1}^\Gamma \phi_i^\tau \right)$, where $\Gamma$ is the number of trade periods, and $N$ is the size of the network. Figure 8 illustrates the convergence of the cumulative deviation between the true and estimated Shapley values for a network of size 5, over a period of 100 trades. The gross absolute cumulative differences drop rapidly, underscoring the practicality of the proposed algorithm in practical trade scenarios.

**9. Concluding Remarks**

The unprecedented growth of the Internet has given rise to rapidly expanding activities in all areas of information creation, distribution, sharing and usage. The substantial externalities in all types of World Wide Web participants ranging from content creators to access providers and end users are increasingly straining the basic infrastructure of the Internet. The notion of DCE is gaining increasing traction as a viable market-based approach to address the effective usage of the infrastructure resources for realizing various value-added services on the Internet. As high-end computing systems evolve into decentralized architectures, distributed resource ownership and consequential trading become viable. In this paper we model a DCE for storage provisioning as a CPN that supports the demand from end users to effectively obtain web content and other services from SP. Classic examples of such demand range from corporate Internet subscribers contracting specific Internet capacity for a stretch of time from SPs for
applications such as video teleconferencing to individual subscribers who may wish to buy capacity for audio/video streaming for home entertainment. Other examples include selective services where an SP could offer a prioritized faster access to chosen websites to a customer for a fee using its local storage capabilities. Such services could be very important to business customers like stockbrokers and investment bankers who may need fast access to selected websites from their mobile devices and/or relatively slow network connections.

We develop the foundations of the market mechanisms that would enable SPs to meet volatile demands for their services through bilateral and multilateral capacity trading. We show using a CPN trading model that each SP would gain by agreeing to buy/sell the additionally needed/excess capacities as market conditions dictate. Such trading is particularly advantageous when the resources are relatively expensive to build and whose non-availability could lead to higher customer churn rates. This study demonstrates the practical business viability of a cooperative CPN market. Consequently, we can see a rapid emergence of the CPN market with several innovative business models. The centralized approach utilized in the current study can be fruitfully extended to consider a decentralized approach where the transactions could be modeled as a network of auctions.

References


PTR, USA.


http://www-db.stanford.edu/peers/


Figure 1: Concavity in the Experimental Sites

Figure 2: Market Maker Mechanism

Figure 3: Total and Average Surplus

Figure 4: Network Size Effect

Figure 5: Sample Size Requirement
Table 1: Comparative Volatility Measures

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Table 2: Time Stability of Discount Factors