QB-1 Yes. Interest is a form of rent. It is a fee for the use of money.

QB-2 Simple interest is interest computed on only the original principal or face amount of a note or other obligation. Compound interest is interest computed on the sum of the principal amount plus all unpaid, accrued interest accumulated to the beginning of the current period.

QB-3 The future amount of $1 is the amount that a single sum of $1, plus the compound interest earned on the single sum, will total at a specific date. The present value of a single sum of $1 due in the future is the present worth of $1 discounted back to the present, referred to as time point zero.

QB-4 The present value of $1 is the present value today of a single future investment of $1. The present value of an ordinary annuity of $1 is the present value today of a series of equal rents of $1 each made at equal intervals of time in the future. The basic difference, therefore, is that the present value of $1 involves only one future payment or withdrawal, whereas the present value of an ordinary annuity of $1 involves more than one future payment or withdrawal.

QB-5 The future amount of $1 is the amount which a single sum of $1, plus the compound interest earned on the single sum, will total at a specific future date. The future amount of an ordinary annuity of $1 is the amount to which a series of rents of $1 each, plus the compound interest on those rents, will total immediately after the last rent in the series is deposited. The basic difference, therefore, is that the future amount of $1 deals with only one rent, whereas the future amount of an ordinary annuity of $1 deals with more than one equal rent.

QB-6 The present value of an ordinary annuity of $1 is the value today of a series of equal rents of $1 discounted back to the present. The future amount of an ordinary annuity of $1 is the amount to which a series of rents of $1 each, plus compound interest on those rents, will total immediately after the last rent in the series is deposited. The basic difference is the point in time at which the value is measured. Present value measures the value before the first rent, and future value measures the value after the last rent.
| a. | 10% | 4   |
| b. | 4   | 8   |
| c. | 1-1/2 | 24 |

Anytime that a business manager needs to compare future sums to present sums, he or she must remove the time value of money before making any valid comparison. Otherwise, the manager would be comparing apples with oranges. Specific examples include decisions about purchasing or leasing property, plant, and equipment. Loan managers must know compound interest in order to instruct others about the proper valuation of assets and liabilities. Accountants must know compound interest to do the accounting jobs requiring it; but more importantly, they will be asked to provide relevant compound interest information to nonaccounting managers to enable them to make better decisions.

**SOLUTIONS TO EXERCISES**

**EB-9  Compound Interest Concepts**
**LG 2, 3**

1. b (less than)
2. a (greater than)
3. a (greater than)
4. b (less than)

**EB-10 Future Amount of a Single Investment**
**LG 2**

1.

Invest $8,000 now.

What will be the future amount on this date?


Interest at 12% is compounded annually.

\[ f = 8,000 \times \text{table factor (Table B-1) for } f_{n=5,i=12\%} \]

\[ = 8,000 \times 1.762 = $14,096 \]

* Shorthand manner of stating values for \( n \) and \( i \).
2. Invest $5,250 now.

What will be the future amount on this date?

Jan. 1, 2011
July 1, 2011
Jan. 1, 2012

Interest at 5% is compounded semiannually.

\[ f = 5,250 \times \text{table factor (Table B-1) for } f_{n=2, i=5\%} \]

\[ = 5,250 \times 1.103 = 5,790.75 \]

3. Invest $8,000 now.

What will be the future amount on this date?

Jan. 1, 2011
Jan. 1, 2012
Jan. 1, 2013
Jan. 1, 2014
Jan. 1, 2015
Jan. 1, 2016
Jan. 1, 2017

Interest at 4% is compounded quarterly.

\[ f = 8,000 \times \text{table factor (Table B-1) for } f_{n=24, i=4\%} \]

\[ = 8,000 \times 2.563 = 20,504 \]
EB-11 Present Value of a Single Sum
LG 3

1.

What is the present value on this date?

The future amount is $19,450.


Interest at 9% is compounded annually.

\[ p = 19,450 \times \text{table factor (Table B-2) for } p_{n=3,i=9\%} \]
\[ = 19,450 \times 0.772 = 15,015.40 \]

2.

What is the present value on this date?

The future amount is $7,000.


Interest at 2-1/2% is compounded quarterly.

\[ p = 7,000 \times \text{table factor (Table B-2) for } p_{n=10,i=2-1/2\%} \]
\[ = 7,000 \times 0.781 = 5,467 \]
EB-11  (continued)

3.

What is the present value on this date?

<table>
<thead>
<tr>
<th>Date</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>July 1, 2011</td>
<td>$15,000</td>
</tr>
</tbody>
</table>

The future amount is $15,000.

Interest at 14% is compounded annually.

\[ p = 15,000 \times \text{table factor (Table B-2)} \text{ for } p_{n=10,i=14\%} \]

\[ = 15,000 \times 0.270 = \text{\$4,050} \]

EB-12  Future Amount of an Annuity

LG 4

What is the future amount on this date?

<table>
<thead>
<tr>
<th>Date</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>July 1, 2011</td>
<td>$500</td>
</tr>
<tr>
<td>Jan. 1, 2012</td>
<td>$500</td>
</tr>
<tr>
<td>Jan. 1, 2013</td>
<td>$500</td>
</tr>
<tr>
<td>Jan. 1, 2014</td>
<td>$500</td>
</tr>
<tr>
<td>Jan. 1, 2015</td>
<td>$500</td>
</tr>
<tr>
<td>Jan. 1, 2016</td>
<td>$500</td>
</tr>
<tr>
<td>Jan. 1, Apr. 2016</td>
<td>$500</td>
</tr>
</tbody>
</table>

Interest at 2-1/2% is compounded quarterly.

\[ F_o = 500 \times \text{table factor (Table B-3)} \text{ for } F_{o,n=20,i=2-1/2\%} \]

\[ = 500 \times 25.545 = \text{\$12,772.50} \]
EB-13 Present Value of an Annuity
LG 5

a.

What is the present value on this date?  

<table>
<thead>
<tr>
<th>$2,500</th>
<th>$2,500</th>
<th>$2,500</th>
<th>$2,500</th>
<th>$2,500</th>
</tr>
</thead>
</table>

5 rents of $2,500

Interest at 10% is compounded annually.

\[ P_o = 2,500 \times \text{table factor (Table B-4)} \text{ for } P_{o_{n=5,i=10\%}} \]

\[ = 2,500 \times 3.791 = \$9,477.50 \]

b.

What is the present value on this date?

<table>
<thead>
<tr>
<th>$800</th>
<th>$800</th>
<th>$800</th>
<th>$800</th>
<th>$800</th>
</tr>
</thead>
</table>

5 rents of $800

Interest at 5-1/2% is compounded semiannually.

\[ P_o = 800 \times \text{table factor (Table B-4)} \text{ for } P_{o_{n=5,i=5-1/2\%}} \]

\[ = 800 \times 4.270 = \$3,416 \]
Future Amount Issue

The $40,000 is the principal sum or present value.

\[ f = $40,000 \times \text{table factor (Table B-1) for } f_{n=36,i=1-1/2\%} \]
\[ = $40,000 \times 1.709 = $68,360 \]

Calculating Future Amount

Amount owed in three months:

\[ f = $700 \times \text{table factor (Table B-1) for } f_{n=3,i=1-1/2\%} \]
\[ = $700 \times 1.046 = $732.20 \]

Interest at 1-1/2\% is compounded monthly.

Amount owed in six months (diagram is not drawn):

\[ f = $700 \times \text{table factor (Table B-1) for } f_{n=6,i=1-1/2\%} \]
\[ = $700 \times 1.093 = $765.10 \]

Amount owed in one year (diagram is not drawn):

\[ f = $700 \times \text{table factor (Table B-1) for } f_{n=12,i=1-1/2\%} \]
\[ = $700 \times 1.196 = $837.20 \]
EB-16  Calculating Required Current Deposit
LG 3

Present value of a single sum at 2.5% for 16 periods (Table B-2).

\[ 40,000 \times 0.674 = 26,960 \]

EB-17  Calculating Required Investment for Retirement
LG 3

Present value of a single sum at 8% and 10% for 40 periods (Table B-2).

\[ 1,000,000 \times 0.046 = 46,000 \quad 1,000,000 \times 0.022 = 22,000 \]

EB-18  Use of Future Amount of $1 Tables to Calculate Needed Present Value
LG 2, 3  Factors

1. \( p = \frac{1}{f} \).

2. \( P_{n=36,i=1.1/2\%} = \frac{1}{1.709} = 0.585 \)

Once this is determined, the present value as at time period zero of $8,000 due 36 months from now is:

\[ p = 8,000 \times \text{table factor determined above for } P_{n=36,i=1.1/2\%} \]

\[ = 8,000 \times 0.585 = 4,680 \]

EB-19  Calculating Future Amount of an Annuity
LG 4

What is the future amount on this date?

5 rents of $4,000

\[ \begin{array}{cccccc}
\text{Dec. 31,} & \text{Dec. 31,} \\
2011 & 2015 \\
\times & \times & \times & \times & \times \\
$4,000 & $4,000 & $4,000 & $4,000 & $4,000 \\
\end{array} \]

Interest at 9% is compounded annually.

\[ F_o = 4,000 \times \text{table factor (Table B-3 for } F_{o=5,i=9\%} \]

\[ = 4,000 \times 5.985 = 23,940 \]
EB-20  Savings to Make a Major Purchase
LG 4

Future value of an annuity at 4.0% for 10 rents (Table B-3).

\[ 8,000 \times 12.006 = 96,048 \]

EB-21  Retiring a Debt
LG 4

Future value of an annuity at 6% for 12 rents (Table B-3).

\[ \text{Rent} \times 16.870 = 500,000 \]
\[ \text{Rent} = 29,638.41 \]

EB-22  College Living Expenses
LG 5

Present value of an annuity at 1.5% for 24 rents (Table B-4).

\[ 1,000 \times 20.030 = \text{\underline{20,030}} \]

EB-23  College Tuition
LG 6

The present value factor for an annuity due of four rents at 8% is equal to the present value factor of an ordinary annuity of three rents (Table B-4) plus 1.0. Thus, the present value factor is as follows:

\[ 2.577 + 1.0 = 3.577 \]

Present value of an annuity due at 8% for four rents.

\[ 10,000 \times 3.577 = \underline{35,770} \]

EB-24  Retirement Planning
LG 5

Present value of an ordinary annuity at 10% for 25 rents (Table B-4).

\[ 50,000 \times 9.077 = 453,850 \]
EB-25  Pre-retirement Planning
LG 3, 5

This exercise requires two steps. First we must determine how much is needed at retirement. This is the present value of an annuity at 10% for 30 rents. Then we must determine the deposits that will be required to accumulate the amount determined in step one. This is the future value of an annuity at 9% for 40 rents.

\[ \$80,000 \times 9.427^* = \$754,160 \]

Rent \( \times 337.882^\dagger = \$754,160 \)

Rent = \$2,232 (Required annual deposit)

^*Table B-4.
^\dagger Table B-3.

SOLUTIONS TO PRACTICE CASES

Practice Case 1

Determining Which Gift to Accept
LG 3, 5

Requirement 1.

a. Present value of \$10,000 cash is \$10,000.

b. Present value of an annuity of \$3,200 discounted at 14%:

\[ P_o = \$3,200 \times \text{table factor (Table B-4) for } P_{o=5,i=14\%} \]
\[ = \$3,200 \times 3.433 = \$10,985.60 \]

c. Present value of a single sum of \$20,000 discounted for 5 years:

\[ p = \$20,000 \times \text{table factor (Table B-2) for } P_{n=5,i=14\%} \]
\[ = \$20,000 \times 0.519 = \$10,380 \]

Ranking in order of highest present value:

b, c, a

Based on this ranking, the student should choose the \$3,200 at the end of each year for five years.

B-10
Practice Case (continued)

Requirement 2.

a. Present value of $10,000 cash is $10,000.

b. Present value of an annuity of $3,200 discounted at 7%:

\[ P_o = 3,200 \times \text{table factor (Table B-4)} \text{ for } P_{o,m=5,i=7\%} \]
\[ = 3,200 \times 4.100 = 13,120 \]

c. Present value of a single sum of $20,000 discounted for 5 years:

\[ p = 20,000 \times \text{table factor (Table B-2)} \text{ for } p_{n=5,i=7\%} \]
\[ = 20,000 \times 0.713 = 14,260 \]

The ranking is now c, a, b.

Requirement 3.

The ranking is the different in the preceding cases. With different interest rates and time periods, there could be a different ordering. This would be caused by the impact of the discount factor on the present value. Notice that as the interest rate decreases the difference between the choices becomes larger.

Practice Case 2

Determining Which Gift to Accept
LG 3,5

a. Present value of $16,000 cash is $16,000.

b. Present value of a single sum of $45,000 discounted for 10 years at 12%:

\[ p = 45,000 \times \text{table factor (Table B-2)} \text{ for } p_{n=10,i=12\%} \]
\[ = 45,000 \times 0.322 = 14,490 \]

c. Present value of a single sum of $50,000 discounted for 12 years at 12%:

\[ p = 50,000 \times \text{table factor (Table B-2)} \text{ for } p_{n=12,i=12\%} \]
\[ = 50,000 \times 0.257 = 15,420 \]
Practice Case 2  (continued)

The *most beneficial* offer is the offer to receive cash of $16,000 now. It has the highest present value. In other words, it could be invested now and earn enough interest to accumulate a fund that would exceed the other two offers in 10 and 12 years.

The *second best* offer is alternative c. This has the second highest present value. For the reason stated above, this amount could be invested when it is received and earn more than alternative b. The risk with this alternative is that it is dependent on the uncle’s death.

The *least beneficial* offer is alternative b. This alternative is the lowest.